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A systematic analysis of new physics effects in spin correlations in $q\overline{q} \to t\overline{t}$

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ABSTRACT: I show that there are eight new physics operators that contribute to top pair production and subsequent decay in interference with the leading SM process at the Tevatron and estimate their expected statistical inaccuracies. All fully differential cross sections are calculated in the 6-particle phase space. The observation of angular correlations dramatically improves the accuracy of the extraction.

KEYWORDS: Standard Model, Beyond Standard Model, Technicolor and Composite Models.

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1. Introduction

Until high energy experiments achieve the neccesary energy and sensitivity to discover new particles in direct production, any experimental progress beyond the Standard Model (SM) should go through the observation of loop effects in precision measurements. These new effects can be parametrized using effective Lagrangians where coefficients of higher dimensional operators carry all the information on new physics. In this paper I ask how accurately one can measure these couplings in proton-proton collisions at presently available energies.

The heavy mass of the top quark allows us to speculate that new physics could show up more easily in the couplings of the top quark then of any other SM particle. The most obvious place do such precision measurements is the Fermilab Tevatron, where at Run 2 an estimated 20,000 top pairs will be produced. In this paper I am looking for small new operators, generated in many theories with strong electroweak physics, large extra dimensions, etc. The succes of the SM leads us to expect that these contributions are small, and this in turn allows us to keep only the interference with the leading SM process, top pair production through $q\overline{q} \to g \to t\overline{t}$. The emerging restriction singles out eight different operators and these can be disentengled once we make use of the information residing in the spin state of the produced top quarks. This spin information is in turn translated into the angular distribution of the decay products of the two $t, \bar{t}$ quarks.

The effect of new physics operators in top pair production has been repeatedly studied in the literature [1]–[5]. Most of these investigations look at top transverse momentum and invariant $t\bar{t}$ distributions in addition to effects in the total cross section. In the following I argue that this approach looses most of the information present in the events because it sums over top polarization states. The top quark decays into $t \to Wb$ so quickly that its spin state is not washed out by strong interactions. In refs. [2, 5] the use of particular asymmetries helps to incorporate at least some of this information. We will see however, that the full use of angular distributions significantly increases the accuracy.

The first attempt in this direction was taken in [6], where the effect of the top chromomagnetic moment on these distributions was analyzed. We now extend this analysis for all operators that can contribute, up to dimension 6. In fact, there is little reason, other than theoretical prejudice based on adherence to particular sorts of models, to expect that any of the operators in the set should be distinctly larger than others. For example, the dimension 5 chromomagnetic moment κ is produced at $\mathcal{O}(1/\Lambda^2)$, similarly to dimension 6 operators (Λ is the scale of the new physics.) In this paper I purposefully avoid any reference to the expected relative size of the new couplings.

In what follows I classify the observable contributions in two cases: (i) when the electroweak (EW) symmetry breaking sector contains only a light Higgs doublet, and (ii) when all new fields are heavy. In the former case we can write an effective Lagrangian where the fields are in a linear representation of the EW group, while in the latter case we must use a nonlinear representation. The approach is thus quite general and should only break down if there are new resonances in the below-1 TeV region that interact significantly with the top quark (such as top pions, for example.) We write down the new operators in both cases and conclude that this experiment cannot tell apart the two scenarios.

The paper is organized as follows. In section 2 I first discuss the relationship of the two representations, and write down a basis of those new operators that contain at least one top field. Then I find the corrections to the $gt\bar{t}$ vertex, to top decay in the tWb vertex and 4-quark vertices: two CP violating and six CP conserving operators. I proceed in section 3 by calculating the contribution of each of these new interactions to the $t\bar{t}$ spin density matrices and to top decay, and compute their contribution to the fully differential cross section in the 6-particle phase space. These contributions can be used for comparing experimental distributions to disentangle each contribution.

It is important to note that one does not need to reconstruct the probability of events in the many-dimensional phase space from the measurement. That would be impossible with the event numbers involved (on the order of several thousand). One

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can instead estimate the new couplings, using for example a Maximum Likelihood method. We perform such an analysis in section 4, using Monte-Carlo-generated data according to the SM distribution. This analysis tells us the attainable statistical accuracy of the estimation.

I sum the results in the Conclusion and discuss the discovery potential of TeVatron Run 2 at Fermilab. I find that new physics at a scale order TeV can be generically detected and that all-hadronic decay modes of the top quarks may play an important role, the complexity of these events notwithstanding.

2. Linear and nonlinear representations: parametrization of new physics in interference

We are interested in effective lagrangians controlling top pair production and subsequent decays with or without a single SM Higgs field, where all other physics resides in the higher dimensional operators. Without including any model dependent "prejudice", we should proceed by writing down all possible operators at each dimension level and look for their experimental consequences.

When a SM Higgs boson is present in the lagrangian, the underlying $\mathrm{SU}(2)\times\mathrm{U}(1)$ symmetry imposes restrictions on the possible form of the new operators. We are free to choose the fields in a linear representation of the gauge group and implement spontaneous electroweak breaking by introducing a Higgs vacuum expectation value. The introduction of a Higss v.e.v however induces lower dimensional new operators of a non-gauge invariant form.

It may well happen that the SM Higgs is very heavy or is not present at all. In the absence of light resonances an effective Lagrangian still make sense, but in that case its field content does not allow us to put all the fields in closed linear representations. Because we are supposing that all additional fields are heavy, we are allowed to put the SM fields, including the three eaten-up Goldstone bosons, in a nonlinear representation of the gauge group and require gauge invariance of the effective lagrangian. [7]

It has been shown [8], however, that the additional restrictions by imposing this nonlinearly realized symmetry are exactly compensated by the additional freedom in the choice of the gauge. Note that the presence of unphysical degrees of freedom in a generic gauge (such as longitudinal and timelike components of the gauge bosons) lets us write down additional operators containing these fields. In the unitary gauge all these nonphysical fields fall away and the net result is that all restrictions on the new operators from gauge invariance disappear. The gauge non-invariant operators allowed this way will be similar to those in the linear representation that contain no Higgs fields.

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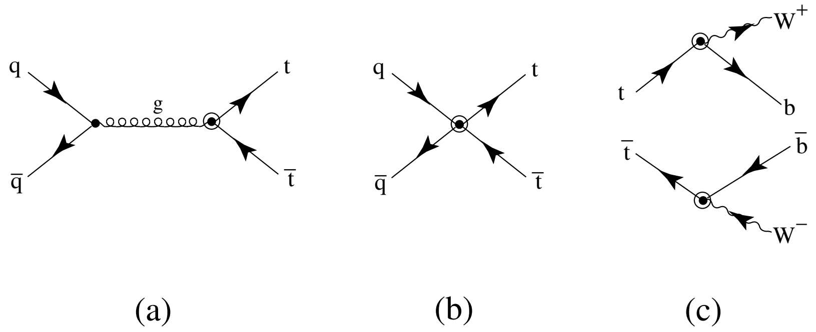

Figure 1: The only types of new top physics processes that contribute to $q\overline{q} \to t\overline{t}$ in interference.

In this section I write down, in both cases, all the new physics operators that can effect the $q\overline{q} \to t\overline{t}$ process, up to dimension 6. There is a huge number of such operators. However, one may impose the following restrictions on those that may give appreciable contributions:

- The use of an effective lagrangian presupposes that the corrections are small, therefore we are justified to keep only the interference between new physics operators and SM processes.

- We keep new physics only in vertices that involve at least one top quark field. Other new physics, including that in light quark interactions, can presumably be seen easier in other processes.

- We drop those new operators that pick up a weak coupling from an unmodified SM vertex (such as corrections to the $Zt\overline{t}$ vertex, contributing to $q\overline{q} \to Z \to t\overline{t}$).

- We drop all loop corrections.

With the above criteria some combinatorics shows that the only contributions come from (a) corrections to the $gt\overline{t}$ vertex, (b) chirality preserving 4-quark operators of the color octet type, $\left(\overline{q}\,\Gamma_{V,A}\frac{\lambda^a}{2}q\right)\left(\overline{t}\,\Gamma_{V,A}\frac{\lambda^a}{2}t\right)$ where the light quarks $q, \overline{q}$ are must be of the same flavor to interfere with the SM process, and (c) corrections to the tWb vertex in top decay (see figure 1).

The linear set of 2-quark (plus boson fields) operators have been classified extensively in the literature [9, 2, 10]. Any complete prediction of the contributions makes sense with reference to a complete set of operators, because any partial list will tacitly imply a particular choice of operators that have zero contribution. For this reason I provide in appendix A a list of the operator basis I am using. This list completely agrees with that of ref. [9] when all CP-odd operators are dropped. In appendix B I also provide the corresponding operators in the nonlinear set.

The contributions we find to the $gt\bar{t}$ vertex are only:

$$-g_s \frac{\kappa}{4m_t} \bar{t}\, \sigma^{\mu\nu} \frac{\lambda^a}{2} G^a_{\mu\nu}\, t - g_s \frac{\tilde{\kappa}}{4m_t} \bar{t}\, \sigma^{\mu\nu} \frac{\lambda^a}{2} {}^{*}G^a_{\mu\nu}\, t\,, \tag{2.1}$$

where the nonlinear set contributes $\kappa = \kappa_g^{33}$ and $\tilde{\kappa} = \tilde{\kappa}_g^{33}$, while the linear set contributes $\kappa = -4m_t \frac{v}{\sqrt{2}} \operatorname{Re} b_{10}^{33}$ and $\tilde{\kappa} = 4m_t \frac{v}{\sqrt{2}} \operatorname{Im} b_{10}^{33}$ (for the detailed meaning of the r.h.s. see the appendix). Both κ and $\tilde{\kappa}$ are required to be real by Hermiticity. The absence of the $\mathcal{O}(p^2)$ terms in the expansion of the $gt\bar{t}$ form factors seems rather strange. These operators have been turned into 4-quark operators by the use of the equations of motion.

Next we look at corrections to top decay. We find the following three operators that seem to contribute to the tWb vertex:

$$\begin{aligned}
&\frac{c_1}{m_t} \frac{g}{\sqrt{2}} W^\mu\, \bar{t}_R\, i \overleftrightarrow{\partial}_\mu\, b_L + h.c. && c_1 = \begin{cases} -\dfrac{\kappa'^{33}_{LW}}{4} & \text{(nonlinear)} \\[2ex] \dfrac{m_t}{2} \dfrac{v}{\sqrt{2}}\, \overline{b_2^{33}} & \text{(linear)} \end{cases} \\[3ex]
-&\frac{c_2}{m_t} \frac{g}{\sqrt{2}} W_{\mu\nu}\, \bar{t}_R\, \sigma^{\mu\nu} b_L + h.c. && c_2 = \begin{cases} \dfrac{\kappa_W^{33} + i\,\tilde{\kappa}_W^{33}}{4} & \text{(nonlinear)} \\[2ex] -m_t\, \dfrac{v}{\sqrt{2}}\, \overline{b_9^{33}} & \text{(linear)} \end{cases} \\[3ex]
i&\frac{c_3}{m_t^2} \frac{g}{\sqrt{2}} \left(\partial^\mu W^\nu + \partial^\nu W^\mu\right) \bar{t}_L\, \gamma_\mu\, i \overleftrightarrow{\partial}_\nu\, b_L + h.c. && c_3 = \begin{cases} -i\rho_{LW} & \text{(nonlinear)} \\[2ex] \quad 0 & \text{(linear)} \end{cases} \quad .
\end{aligned} \tag{2.2}$$

In the following we will see however, that only three real linear combinations of these operators contribute to the differential cross section; the operator with c_3 does not contribute at all. It is not unexpected to see fewer operators contribute to top decay because of the kinematical restrictions. Only the CP-even combination

$$\mu_1 = 2\frac{m_t^2 - m_W^2}{m_t^2} \operatorname{Re}\left(2c_2 + c_1 + c_3\right) \tag{2.3}$$

and the CP-odd combination

$$\mu_2 = 2\frac{m_t^2 - m_W^2}{m_t^2} \operatorname{Im}\left(2c_2 + c_1 + c_3\right) \tag{2.4}$$

affect the differential cross sections, while the CP-even

$$\mu_0 = -2\frac{m_t^2 - m_W^2}{m_t^2} \operatorname{Im}\left(c_1 + c_3\right) \tag{2.5}$$

only changes the total rate by an overall factor of $(1+\mu_0)$. Because this last factor would show up only in the total top decay rate, it is irrelevant for our investigation and I drop it in the following. Note that in a similar manner I ignored the contribution of a $\sim W^\mu \,\bar{t}_L\, \gamma_\mu\, b_L$ operator which would only modify V_{tb}.

Finally, we turn to the effect of 4-quark operators. These are the same in both set and are severely restricted by the above criteria:

$$\frac{g_s^2}{4m_t^2} \times \left\{ c_{VV} \times \left(\bar{q}\gamma_\mu \frac{\lambda^a}{2} q\right) \left(\bar{t}\gamma^\mu \frac{\lambda^a}{2} t\right) + c_{VA} \times \left(\bar{q}\gamma_\mu \frac{\lambda^a}{2} q\right) \left(\bar{t}\gamma^\mu\gamma^5 \frac{\lambda^a}{2} t\right) + \right. \tag{2.6}$$
$$\left. + c_{AV} \times \left(\bar{q}\gamma_\mu\gamma^5 \frac{\lambda^a}{2} q\right) \left(\bar{t}\gamma^\mu \frac{\lambda^a}{2} t\right) + c_{AA} \times \left(\bar{q}\gamma_\mu\gamma^5 \frac{\lambda^a}{2} q\right) \left(\bar{t}\gamma^\mu\gamma^5 \frac{\lambda^a}{2} t\right) \right\},$$

where the coefficients $c_{VV}, c_{VA}, c_{AV}, c_{AA}$, must all be real ($q = u, d, s, \ldots$ is the light quark). The selection of these operators has been pressed on us by the required presence of interference with the SM process. However, our operators can be embedded into the set of custodial invariant 4-fermion operators discussed in [3]. Therefore, we find no direct restriction on our operators from custodial symmetry. I separated off a factor of g_s^2 for easy comparison (this factor does not need to reflect any strong interaction physics.)

This completes the list of all contributing operators. Their size remains remarkably unrestricted by previous experiments and by theoretical arguments. None of them contributes linearly at tree level to the electroweak precision parameters. Consistency of the effective theory requires that their size cannot significantly exceed $\left(\frac{2m_t}{\Lambda}\right)^2$, but the new physics scale Λ is unknown. All we know for sure is that they must be less than unity, and this fact justifies throwing away all contributions that are quadratic in the new couplings.

It is now a straightforward but tedious calculation to compute the contribution of each of these operators to the differential cross section, a calculation to which we now turn.

3. New physics contributions to the differential cross sections

In this section I derive the most important result of this paper: the contribution of each of the operators in (2.1), (2.2), (2.6) to the fully differential cross section comprising $t\bar{t}$ pair production and decay.

The SM process is shown in figure 2. By D I designate the down-type quark in the decay. In (semi)leptonic decays it should be replaced by a charged lepton. In the following I will keep, for the economy of the notation, p_D for the lepton momentum; I well proceed similarly with $\overline{D}, U$ and $\overline{U}$.

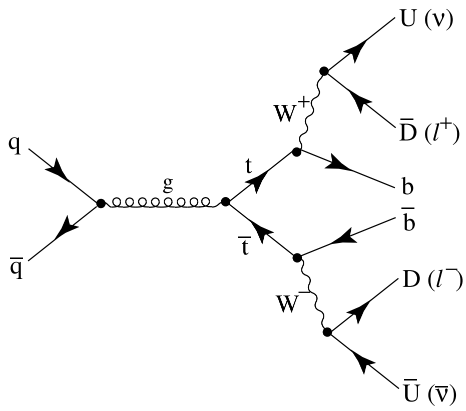

Figure 2: The full cascade of $t\bar{t}$ production and decay in the Standard Model. The decay products of the W^+ are an up-type $U = u, c$ quark (or a neutrino) and a down-type antiquark $\overline{D} = \overline{d}, \overline{s}$ (or a positively charged lepton, e^+ or μ^+).

A Breit-Wigner formula is applicable to the amplitudes when the t and $\bar{t}$, as well as the $W^\pm$ are on shell:

$$\mathcal{M}_{q\bar{q}\to 6} = \sum_{s,\bar{s}} \mathcal{M}_{t\to 3}(s) \left(\frac{\pi\,\delta\,(p_t^2 - m_t^2)}{m_t \Gamma_t}\right)^{1/2} \mathcal{M}_{q\bar{q}\to t\bar{t}} \left(\frac{\pi\,\delta\,(p_{\bar{t}}^2 - m_t^2)}{m_t \Gamma_t}\right)^{1/2} \mathcal{M}_{\bar{t}\to 3}(\bar{s})\,. \tag{3.1}$$

This equation allows us to separate the physics into top production, described by the density matrix $\rho_{s\bar{s},s'\bar{s}'}$ and the decay represented by $T^{(t)}_{ss'}$ and $T^{(\bar{t})}_{\bar{s}\bar{s}'}$:

$$\frac{g_s^4}{9} \times \rho_{s\bar{s},s'\bar{s}'} = \overline{\sum} |\mathcal{M}_{q\bar{q}\to t\bar{t}}|^2\,, \tag{3.2}$$

averaged for the incoming spin and color, summed for outgoing color (but the $t, \bar{t}$ spins are kept fixed).

The decay of the $t, \bar{t}$ quarks is described by

$$\begin{aligned}
\sum \overline{\mathcal{M}_{t\to 3}\,(s')}\mathcal{M}_{t\to 3}\,(s) &= \frac{T^{(t)}_{ss'}}{|p_W^2 - m_W^2|^2} \Longrightarrow \left(\frac{\pi\,\delta\,(p_W^2 - m_W^2)}{m_W \Gamma_W}\right) \times T^{(t)}_{ss'} \\
\sum \overline{\mathcal{M}_{\bar{t}\to 3}\,(\bar{s}')}\mathcal{M}_{\bar{t}\to 3}\,(\bar{s}) &= \frac{T^{(\bar{t})}_{\bar{s}\bar{s}'}}{|p_{\overline{W}}^2 - m_W^2|^2} \Longrightarrow \left(\frac{\pi\,\delta\,(p_{\overline{W}}^2 - m_W^2)}{m_W \Gamma_W}\right) \times T^{(\bar{t})}_{\bar{s}\bar{s}'}\,.
\end{aligned} \tag{3.3}$$

In the decay contribution I separate out factors that do not receive contributions from new physics operators,

$$\begin{aligned}
T^{(t)}_{ss'} &= \left(\frac{g}{\sqrt{2}}\right)^4 |V_{U\overline{D}}|^2 \times 8\,(p_b \cdot p_U)(p_t \cdot p_{\overline{D}}) \times \tau^{(t)}_{ss'} \\
T^{(\bar{t})}_{\bar{s}\bar{s}'} &= \left(\frac{g}{\sqrt{2}}\right)^4 |V_{\overline{U}D}|^2 \times 8\,(p_{\bar{b}} \cdot p_{\overline{U}})(p_{\bar{t}} \cdot p_D) \times \tau^{(\bar{t})}_{\bar{s}\bar{s}'}\,.
\end{aligned} \tag{3.4}$$

The six particle phase space becomes manageable when we describe the momenta of the three t decay products in the t frame and the three $\bar{t}$ decay products in the $\bar{t}$ frame. Denoting the three-direction of the $W(\overline{W})$ by $\mathbf{w}(\overline{\mathbf{w}})$ and the three-direction of the $\overline{D}(D)$ by $\mathbf{n}(\overline{\mathbf{n}})$in these frames, we find for the differential cross section

$$
\begin{aligned}
d\sigma = & \frac{\alpha_s^2 \beta^2}{9 \times 2^{24}\pi^9} \frac{(m_t^2 - m_W^2)^2}{m_Z^4 m_W^2 \Gamma_t^2 \Gamma_W^2 (m_t^2 + m_W^2)^4} \times \left[4 \left(\frac{g}{\sqrt{2}} \right)^4 \times 2 \frac{m_t^2 m_W^2}{m_t^2 + m_W^2} \right]^2 \times \\
& \times |V_{U\overline{D}}|^2 |V_{\overline{U}D}|^2 \times d\Omega_t^{(\mathrm{ZMF})} \times d\Omega(\mathbf{n}) \times \frac{d\Omega(\mathbf{n})}{(1 - \beta_W \mathbf{n} \cdot \mathbf{w})^3} (p_b \cdot p_U) \times d\Omega(\mathbf{n}) \times \\
& \times \frac{d\Omega(\overline{\mathbf{n}})}{(1 - \beta_W \overline{\mathbf{n}} \cdot \overline{\mathbf{w}})^3} (p_{\bar{b}} \cdot p_{\overline{U}}) \times Z_t \,, \qquad (3.5)
\end{aligned}
$$

where $d\Omega_t^{(\mathrm{ZMF})}$ is the solid angle measure of the top direction in the zero momentum frame (i.e. the center of mass frame of the $t\bar{t}$ system), $\beta_W = \frac{m_t^2 - m_W^2}{m_t^2 + m_W^2}$ is the speed of the W in the t frame, and the new physics corrections are all factorized into

$$
Z_t = \sum_{s\bar{s}, s'\bar{s}'} \frac{\rho_{s\bar{s}, s'\bar{s}'}}{4} \times \tau_{ss'}^{(t)} \times \tau_{\bar{s}\bar{s}'}^{(\bar{t})} \,. \qquad (3.6)
$$

The spin states corresponding to the indices $s, \bar{s}$ require careful definition: because we are also using the off-diagonal elements of the density matrix, the relative phase of the spin states must be taken into account. We achieve this using the following conventions. The spin vectors $s, \bar{s}$ lay in the scattering plane and their spatial components are back-to back to each other in the ZMF frame. A vector ξ^μ is chosen normal to the scattering plane, and the for a spin basis I use the so-called "off-diagonal" basis of ref. [11] in which the SM contribution to the density matrix is simple. Our conventions are best understood in the ZMF where the various are

$$
\begin{aligned}
p_q^\mu &= \frac{\sqrt{\hat{s}}}{2} (1|1|0|0)^\mu & p_{\bar{q}}^\mu &= \frac{\sqrt{\hat{s}}}{2} (1|-1|0|0)^\mu \\
p_t^\mu &= \frac{\sqrt{\hat{s}}}{2} (1|\beta\cos\Theta|\beta\sin\Theta|0)^\mu & p_{\bar{t}}^\mu &= \frac{\sqrt{\hat{s}}}{2} (1|-\beta\cos\Theta|-\beta\sin\Theta|0)^\mu \\
\eta^\mu &= \frac{-1}{\sqrt{1-\beta^2\sin^2\Theta}} (\beta\sin\Theta|0|1|0)^\mu & \overline{\eta}^\mu &= \frac{-1}{\sqrt{1-\beta^2\sin^2\Theta}} (\beta\sin\Theta|0|-1|0)^\mu
\end{aligned}
\qquad (3.7)
$$

$$
\begin{aligned}
s^\mu &= \frac{\gamma}{\sqrt{1-\beta^2\sin^2\Theta}} (\beta\cos\Theta|1-\beta^2\sin^2\Theta|\beta^2\sin\Theta\cos\Theta|0)^\mu \\
\bar{s}^\mu &= \frac{\gamma}{\sqrt{1-\beta^2\sin^2\Theta}} (\beta\cos\Theta|-(1-\beta^2\sin^2\Theta)|-\beta^2\sin\Theta\cos\Theta|0)^\mu \\
\xi^\mu &= (0|0|0|1)^\mu \,. \qquad (3.8)
\end{aligned}
$$

Here, β is the speed of the top quark in the ZMF frame, $\gamma = 1/\sqrt{1-\beta^2}$ and Θ is the top scattering angle.

With these definitions the phases of the spin states are fixed by requiring

$$
\begin{aligned}
u_\uparrow \overline{u}_\downarrow &= +\frac{1}{2}(\gamma.p_t + m_t)\ \gamma^5\ \gamma.(\xi + i\eta) \qquad & u_\downarrow \overline{u}_\uparrow &= +\frac{1}{2}(\gamma.p_t + m_t)\ \gamma^5\ \gamma.(\xi - i\eta) \\
v_\uparrow \overline{v}_\downarrow &= -\frac{1}{2}(\gamma.p_{\overline{t}} - m_t)\ \gamma^5\ \gamma.(\xi - i\overline{\eta}) \qquad & v_\downarrow \overline{v}_\uparrow &= -\frac{1}{2}(\gamma.p_{\overline{t}} - m_t)\ \gamma^5\ \gamma.(\xi + i\overline{\eta})\,,
\end{aligned}
\tag{3.9}
$$

where the $\uparrow$ ($\downarrow$) states now have spin projection $+1/2\,(-1/2)$ on the corresponding spin vector.

In the $t(\overline{t})$ frame the purely spatial vectors $\boldsymbol{\eta}, \mathbf{s}, \boldsymbol{\xi}\,(\overline{\boldsymbol{\eta}}, \overline{\mathbf{s}}, \boldsymbol{\xi})$ form right handed orthonormal bases, so that they are used as the respective x, y, z axes to help define the directions of the momenta of the decay products. Although the differential cross section in the six-particle phase space is independent of the choice of the $t, \overline{t}$ spin basis, the choice of the variables does depend on these Cartesian frames.

Given these conventions we can now calculate the ρ and τ matrices in the SM,

$$
\rho^{SM}_{\uparrow\downarrow,\uparrow\downarrow} = \rho^{SM}_{\downarrow\uparrow,\downarrow\uparrow} = 2 - \beta^2 \sin^2\Theta, \qquad \rho^{SM}_{\uparrow\downarrow,\downarrow\uparrow} = \rho^{SM}_{\downarrow\uparrow,\uparrow\downarrow} = \beta^2 \sin^2\Theta,
\tag{3.10}
$$

and all other elements vanish [11]. The decay contributions are, in the notation $\begin{bmatrix} \uparrow\uparrow & \uparrow\downarrow \\ \downarrow\uparrow & \downarrow\downarrow \end{bmatrix}$,

$$
\begin{aligned}
\tau^{(t)}_{SM} &= \begin{bmatrix} 1 + \mathbf{s}\cdot\mathbf{n} & \mathbf{n}\cdot(\boldsymbol{\xi} + i\boldsymbol{\eta}) \\ \mathbf{n}\cdot(\boldsymbol{\xi} - i\boldsymbol{\eta}) & 1 - \mathbf{s}\cdot\mathbf{n} \end{bmatrix} = \begin{bmatrix} 1 + \cos\vartheta & \sin\vartheta e^{i\varphi} \\ \sin\vartheta e^{-i\varphi} & 1 - \cos\vartheta \end{bmatrix}, \\
\tau^{(\overline{t})}_{SM} &= \begin{bmatrix} 1 - \overline{\mathbf{s}}\cdot\overline{\mathbf{n}} & \overline{\mathbf{n}}\cdot(\boldsymbol{\xi} + i\overline{\boldsymbol{\eta}}) \\ \overline{\mathbf{n}}\cdot(\boldsymbol{\xi} - i\overline{\boldsymbol{\eta}}) & 1 + \overline{\mathbf{s}}\cdot\overline{\mathbf{n}} \end{bmatrix} = \begin{bmatrix} 1 - \cos\overline{\vartheta} & \sin\overline{\vartheta} e^{i\overline{\varphi}} \\ \sin\overline{\vartheta} e^{-i\overline{\varphi}} & 1 + \cos\overline{\vartheta} \end{bmatrix}.
\end{aligned}
\tag{3.11}
$$

Substituting (3.10) and (3.11) into (3.6) we find

$$
Z_t^{(SM)} = (\,1 + \cos\vartheta\cos\overline{\vartheta}\,)(1 - \frac{1}{2}\beta^2\sin^2\Theta) + \frac{1}{2}\beta^2\sin^2\theta\sin\vartheta\sin\overline{\vartheta}\cos(\varphi - \overline{\varphi}).
\tag{3.12}
$$

In these expressions I introduced the polar angles in the "off-diagonal" basis. These are defined as[1]

$$
\begin{array}{lll}
\mathbf{n}\cdot\mathbf{s} = \cos\vartheta & \mathbf{n}\cdot\boldsymbol{\xi} = \cos\varphi\sin\vartheta & \mathbf{n}\cdot\boldsymbol{\eta} = \sin\varphi\sin\vartheta \\
\overline{\mathbf{n}}\cdot\overline{\mathbf{s}} = \cos\overline{\vartheta} & \overline{\mathbf{n}}\cdot\boldsymbol{\xi} = \cos\overline{\varphi}\sin\overline{\vartheta} & \overline{\mathbf{n}}\cdot\overline{\boldsymbol{\eta}} = \sin\overline{\varphi}\sin\overline{\vartheta} \\
\mathbf{w}\cdot\mathbf{s} = \cos\chi & \mathbf{w}\cdot\boldsymbol{\xi} = \cos\phi\sin\chi & \mathbf{w}\cdot\boldsymbol{\eta} = \sin\phi\sin\chi \\
\overline{\mathbf{w}}\cdot\overline{\mathbf{s}} = \cos\overline{\chi} & \overline{\mathbf{w}}\cdot\boldsymbol{\xi} = \cos\overline{\phi}\sin\overline{\chi} & \overline{\mathbf{w}}\cdot\overline{\boldsymbol{\eta}} = \sin\overline{\phi}\sin\overline{\chi}
\end{array}
\tag{3.13}
$$

Any corrections to the $gt\overline{t}$ vertex can only show up in ρ while new physics in top decay will affect only the τ's. At this point the astute reader may observe that while the decay contributions affect the distributions in all the observed angles, the

[1] There is a sign difference in the definition of the angles $\varphi, \overline{\varphi}$ that I am using here and the one used in [6]. The difference shows up in the form of the contribution due to κ.

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corrections to top production cannot change the distribution in $\mathbf{w}, \overline{\mathbf{w}}$. This clear distinction is, however, lost when experimental realities are taken into account. The jets produced by the $U, \overline{D}$ quarks are indistinguishable and the required symmetrization brings back the $\mathbf{w}$-dependence.

Now look at the contributions from each of the twelve new physics operators. I quote here only the result of a tedious but straightforward calculation. The contribution of the chromoelectric and chromomagnetic moments into Z_t is

$$\begin{aligned} Z_\kappa &= \kappa \times \left\{2(1+\cos\vartheta\cos\overline{\vartheta})+\beta^2\gamma\sin\Theta\cos\Theta(\sin\vartheta\cos\overline{\vartheta}\sin\varphi+\sin\overline{\vartheta}\cos\vartheta\sin\overline{\varphi})\right\}+ \\ &\quad +\tilde{\kappa}\times\left\{\beta\gamma\,\sin\Theta\,\sqrt{1-\beta^2\sin^2\Theta}\,(\sin\vartheta\cos\overline{\vartheta}\cos\varphi+\sin\overline{\vartheta}\cos\vartheta\cos\overline{\varphi})\right\}, \end{aligned} \tag{3.14}$$

and the four-quark operators contribute

$$\begin{aligned} Z_C &= \\ &= c_{VA}\times\beta\gamma\frac{\gamma(2-\beta^2\sin^2\Theta)\cos\Theta(\cos\vartheta+\cos\overline{\vartheta})-\sin\Theta(\sin\vartheta\sin\varphi+\sin\overline{\vartheta}\sin\overline{\varphi})}{\sqrt{1-\beta^2\sin^2\Theta}}+ \\ &\quad +c_{AA}\times\beta\gamma\left\{2\gamma\cos\Theta(1+\cos\vartheta\cos\overline{\vartheta})-\sin\Theta(\sin\vartheta\cos\overline{\vartheta}\sin\varphi+\sin\overline{\vartheta}\cos\vartheta\sin\overline{\varphi})\right\}+ \\ &\quad +c_{AV}\times 2\gamma^2\sqrt{1-\beta^2\sin^2\Theta}(\cos\vartheta+\cos\overline{\vartheta})+ \\ &\quad +c_{VV}\times 2\gamma^2\left\{(1+\cos\vartheta\cos\overline{\vartheta})(1-\frac{1}{2}\beta^2\sin^2\Theta)+\frac{1}{2}\beta^2\sin^2\Theta\sin\vartheta\sin\overline{\vartheta}\cos(\varphi-\overline{\varphi})\right\}. \end{aligned} \tag{3.15}$$

These contributions, as I explained above, do not depend on the angles $\chi, \phi, \overline{\chi}, \overline{\phi}$. The decay corrections, however, do:

$$\begin{aligned} Z_a &= -\mu_1\times\Bigg\{\Big[\sin\vartheta\sin\chi\cos(\varphi-\phi)+\sin\overline{\vartheta}\sin\overline{\chi}\cos(\overline{\varphi}-\overline{\phi})+ \\ &\qquad +(\cos\vartheta+\cos\overline{\vartheta})(\cos\chi+\cos\overline{\chi})\Big]\times(1-\frac{1}{2}\beta^2\sin^2\Theta)- \\ &\qquad -\frac{1}{2}\beta^2\sin^2\Theta\left[\sin\overline{\vartheta}\sin\chi\cos(\overline{\varphi}-\phi)+\sin\vartheta\sin\overline{\chi}\cos(\varphi-\overline{\phi})\right]\Bigg\}+ \\ &\quad +\mu_2\Bigg\{\left[\sin\vartheta\cos\overline{\vartheta}\sin\chi\sin(\varphi-\phi)+\cos\vartheta\sin\overline{\vartheta}\sin\overline{\chi}\sin(\overline{\varphi}-\overline{\phi})\right]\times \\ &\qquad \times\left(1-\frac{1}{2}\beta^2\sin^2\Theta\right)-\frac{1}{2}\beta^2\sin^2\Theta\Big[\sin\vartheta\sin\overline{\vartheta}(\cos\chi-\cos\overline{\chi})\sin(\varphi-\overline{\varphi})+ \\ &\qquad +\sin\vartheta\cos\overline{\vartheta}\sin\overline{\chi}\sin(\varphi-\overline{\phi})+\sin\overline{\vartheta}\cos\vartheta\sin\chi\sin(\overline{\varphi}-\phi)\Big]\Bigg\}+ \\ &\quad +\mu_1\times\left\{(1+\cos\vartheta\cos\overline{\vartheta})(1-\frac{1}{2}\beta^2\sin^2\Theta)+\frac{1}{2}\beta^2\sin^2\Theta\sin\vartheta\sin\overline{\vartheta}\cos(\varphi-\overline{\varphi})\right\}. \end{aligned}$$

The contribution of the CP conserving μ_1 in the last line is proportional to the SM contribution and only rescales the CKM matrix elements by a factor $(1+\mu_1)$. One can drop this contribution.

The form of the contributions in eqns. (3.14), (3.15), (3.16) constitutes the main result of the present paper. They can be used in an experiment to disentengle the contributions from the various new physics operators. Integrating over the various angles except Θ as required by eqns. (3.5) immediately convinces us that all the information on many of the operators resides in top polarization. Namely, the integration is equivalent to replacing

$$Z_t \to Z_t = \left(1 - \frac{1}{2}\beta^2 \sin^2\Theta\right)\left[1 + \mu_1 + 2\gamma^2 \times c_{VV}\right] + 2\kappa + 2\beta\gamma^2 \cos\Theta \times c_{AA} \,. \quad (3.16)$$

The only couplings that affect the distribution of the events in the $t, \bar{t}$ momenta at all are μ_1, c_{VV} and c_{AA}. Out of these, c_{AA} does not contribute to the total cross section.

4. Sensitivities

The parton-level calculations in section 3 should be confronted with experimental realities. At first sight one would think that the large inaccuracies related to the extraction of parton momenta from jet observables preclude a detailed mapping of the phase space. However, in order to establish the presence of the new physics operators one does not need to reconstruct the full angular distribution. The complicated structure of eqns. (3.14), (3.15), (3.16) does not allow us to select a simple observable that would indicate the presence of new physics. Instead, one should look for the particular types of angular correlations that are similar to those in the above equations. This is most easily done by a Maximum Likelihood estimate. Following this strategy the angular correlations are not spoiled by uncertainties in the jet variables even as large as $\pm 50\%$ or by very large backgrounds [6].

The events fall into three categories, according to the decay mode of the $t, \bar{t}$:

- Dilepton events. In this case two neutrinos are produced and full event reconstruction is extremely hard. The angles we are investigating are probably impossible to extract.

- Semileptonic events. These provide an opportunity to accurately measure the momentum of the charged lepton (i.e. $p_{\overline{D}}$ or p_D), but on the hadronic side the two light quark jets are practically indistinguishable. One must symmetrize the predicted differential cross sections and compare the resulting quantity to the experiment. Fortunately, we will see that this procedure does not entail much information loss. However, the smaller leptonic top decay branching ratio results in smaller statistics compared to the all-hadronic decays.

- All hadronic events. In this case the advantages of high statistics are somewhat compensated by the hardships of relating six jets to six partons. Nevertheless, two SVX tags may identify the $b, \overline{b}$ quarks and requiring that the both tops and W's are on shell leaves a chance for an approximate event reconstruction. One is faced, however, with more complicated ambiguities in this case. There are two pairs of indistinguishable light quarks jets plus the entire t side is hard to distinguish from the $\overline{t}$ side. The former problem does not introduce large uncertainties in the estimation of new physics parameters. As for the latter, we observe that the effect of $t \Leftrightarrow \overline{t}$ replacement amounts to the replacement of $\alpha \leftrightarrow \pi - \overline{\alpha}$ of all the angles in the above formulas for Z_t. Consequently, some of the operators (namely, c_{AA} and c_{AV}) are antisymmetric under this replacement and cancel out from the symmetrized cross sections. The rest however, including both operators in top decay, are symmetric and survive unchanged.

In order to assess the statistical uncertainties involved in the determination of the new physics parameters I performed a Maximum Likelihood estimation of the eight real parameters $\mu_i = \mu_1, \mu_2, \kappa, \tilde{\kappa}, c_{VV}, c_{VA}, c_{AV}, c_{AV}$. I used a Monte Carlo generator to produce events according to the SM distribution at $\sqrt{s} = 2\,TeV$ and built a Maximum Likelihood estimator that finds the most probable values of the μ_i's (the true values are obviously zero.)[2] The fall of the probability by a factor of $e^{-1/2}$ was interpreted as a "$1-\sigma$" interval (in the following I will call its size two times the "accuracy".) In each case the "accuracies" agreed well with the ones found from a second-order expansion of the log-likelihood function, so that the use of a covariance matrix C_{ij} is justified:

$$\ln \mathcal{L}(\mu) = \ln \mathcal{L}_{max} - \frac{1}{2}\mu_i C_{ij} \mu_j \, . \tag{4.1}$$

As a check of the reliability of these estimates, I observed that in each direction in the parameter space when only one μ_i is kept nonzero, the likelihood quickly decreases as μ_i is moved outside the "$1-\sigma$" interval. The true value $\mu_i = 0$ has always been consistent with the "accuracy." In addition, using different event numbers (all above $\mathcal{N} \geq 1200$), I observed that the accuracy is indeed proportional to $\mathcal{N}^{-1/2}$.

The accuracies of the results of this ML estimate are shown in table 1 for 2000 and 20,000 events. The former is relevant for the semileptonic events at the Fermilab Tevatron, while the latter is a hypothetical number shown in order to expose the gain in the accuracy when more events are observed. These numbers immediately confirm that the indistinguishability of the light quark jets does not drastically deteriorate the extraction, contrary to what one might have naïvely expected.

The intricate pattern of correlations induced by the contributions to Z_t tells us that the much of the information on the new operators resides in the spins of the $t, \overline{t}$.

[2] This analysis, an extension to many parameters of the one we used in [6], leads to the "best estimator" $\frac{1}{Z_t^{SM}} \frac{\partial}{\partial \mu_i} Z_t(\mu_i)$. After the publication of [6] I realized that this is the same estimator as the one described in [12], found through a different argument unrelated to Maximum Likelihood.

μ_i	CP	? σ_{tot}	? $t \leftrightarrow \bar{t}$	No Pol.	No Symm.		$U \leftrightarrow \overline{D}$		All Symm.		No Pol.		No Pol., $t \leftrightarrow \bar{t}$	
					$20k$	$2k$	$20k$	$2k$	$20k$	$2k$	$20k$	$2k$	$20k$	$2k$
μ_1	+	yes	yes		1.4	4.2	1.5	**4.6**	1.7	5.1				
μ_2	−		yes		3.4	11	4.4	**14**	5.6	18				
κ	+	yes	yes	yes	1.7	5.8	2.0	**6.8**	2.1	7.2	2.8	27	2.8	27
$\tilde{\kappa}$	−		yes		2.0	6.5	2.6	**8.6**	3.3	11				
c_{VV}	+	yes	yes	yes	0.4	1.2	0.4	**1.2**	0.4	1.2	0.4	1.3	0.4	1.3
c_{VA}	−		yes		0.4	1.1	0.4	**1.2**	0.4	1.3				
c_{AV}	−				0.2	0.8	0.3	**0.9**						
c_{AA}	+			yes	0.3	1.0	0.3	**1.0**			0.4	1.7		
$\kappa - 0.09c_{VV}$	+	yes	yes	yes	1.9	6.2	2.2	**7.6**	2.4	8.4	3.6	29	3.6	29
$c_{VV} + 0.09\kappa$	+	yes	yes	yes	0.4	1.2	0.4	**1.2**	0.4	1.2	0.4	1.9	0.4	1.9

Table 1: The statistical accuracy of the Maximum Likelihood estimate of each parameter μ_i. The numbers mean 100 times the half-width of the "1-sigma" interval. The 2^{nd}, 3^{rd} and 4^{th} columns tell if the operator contributes to the total cross section, whether it survives $t \leftrightarrow \bar{t}$ symmetrization or the integration over all decay angles except Θ (called "No Pol."). The $2k$, $20k$ refer to the number of events considered. "All Symm." denotes symmetrization for $D \leftrightarrow \overline{D}$, $U \leftrightarrow \overline{U}$ and $t \leftrightarrow \bar{t}$. The column in boldface applies directly to semileptonic events at Tevatron Run 2.

When all the angles related to top decay are integrated out and only the variables β and Θ are kept, a reduced amount of information is still available. However, the only operators whose contributions do not vanish in this case are κ, c_{AA} and c_{VV}. We see in table 1 that in the case of c_{AA} and c_{VV}, much information resides in the β, Θ-distributions and these two operators can be observed without measuring the angular correlations of the decay products. In fact c_{VV} represents only a momentum-dependent, $\mathcal{O}(p^2)$ correction to the strong coupling of the top, so its contribution is proportional to that of the SM process times γ^2. This different dependence on the total energy allows that we see this operator in the unpolarized $t\bar{t}$ cross section. In the case of κ, the accuracy drastically deteriorates when we integrate out the polarization information, in accordance with the findings.[3]

The difference in the patterns of angular correlations introduced by each operator leads to the fact that, with one exception, the principal axes of the covariance matrix approximately coincide with the chosen operator basis (i.e. C_{ij} is approximately diagonal.) This means that the angular correlations provide almost independent measurements of each operator in our basis. In the case of κ and c_{VV}, there is an $\mathcal{O}(10\%)$ mixing and the combinations corresponding to the principal axes are shown in the last two rows of table 1.

The accuracies attainable in this way, supposing that systematic uncertainties will not play a drastic role, are generically ± 0.01 to ± 0.1. This can be translated to an observable new physics scale of $\Lambda = \mathcal{O}(6 - 20)m_t$, with an uncertain numerical factor of order unity.[4] The 4-quark operators can be measured with a precision

[3] See [6], see also [1] for the deterioration of the accuracy.

[4] The normalization of the couplings μ_i is so chosen that all mass dimensions are removed at the

of ± 0.01 in semileptonic events, as far as only statistical inaccuracies are taken into account. Two of these, c_{AV} and c_{AA} are theoretically disfavored in many models as they involve an axial vector current of light quarks. The other two, c_{VV} and c_{VA} may however remain unsuppressed as they are related to vector currents of the light quarks. Of course, the actual suppression of each operator is a model-dependent question and should not be addressed in this paper.

5. Conclusions and prospects of discovery

In the above we found that four CP-conserving and four CP-violating new physics operators can contribute to top pair production in interference with $q\bar{q} \to t\bar{t}$: the chromomagnetic and chromoelectric moments $\kappa, \tilde{\kappa}$, two decay corrections μ_0, μ_1 and the four current-current color octet 4-quark operators $c_{VV}, c_{VA}, c_{AV}, c_{AA}$ (averaged over the light quark flavors.) In deriving our set of contributing operators I used the equations of motion, so that some of the physics in the $gt\bar{t}$ couplings may actually show up in the four-quark operators (for example, $G^a_{\mu\nu}\,\bar{t}\frac{\lambda^a}{2}\gamma^\mu D^\nu\, b$ is eliminated.) All these operators arise in both the linear and the nonlinear (chiral) set, so the two cases cannot be told apart in this process. I calculated the contribution of each of these operators to the fully differential cross section $d\sigma(q\bar{q} \to t\bar{t} \to 6\, fermions)$, at tree level, in the approximation when all $t, \bar{t}, W^+, W^-$ are on shell. This is the dominant process of top pair production on the Tevatron at $\sqrt{s} = 2\,TeV$. At higher energies (for example at the LHC) our analysis is not relevant, because other Standard Model processes become important.

Using a maximum likelihood method I estimated the statistical inaccuracy of the measurement of this parameter set. We find that only c_{AA} and c_{VV} can be meaningfully measured without using the information encoded in the directions of the momenta of the six decay products. In addition, the contribution of all operators except c_{VV} to the total cross section vanishes. It is well known that the total cross section is not measured very precisely at the Tevatron, due to inaccurate knowledge of the luminosity. This fact can affect therefore only the extraction of c_{VV} but not that of the other parameters.

The statistical inaccuracies in the case of 2000 semileptonic events (feasible at the Tevatron), generically $\pm\, \mathcal{O}(0.01 - 0.1)$, correspond to new physics on the TeV scale. The actual assessment of the meaning of this accuracy should be performed in the context of particular models, but in any case these accuracies are an order of magnitude better than those previously found without the use of spin information. Although these results seem to be quite robust against large backgrounds or inaccuracies in event reconstruction, it remains to be seen how large is the effect of

scale of $\sim 2\,m_t$. Most $t\bar{t}$ pairs are produced with close to threshold energies, due to the increase of the parton distribution functions at small x, so that m_t is the only energy scale in the problem. However, there remains some $\mathcal{O}(1)$ arbitrariness in the normalization.

asymmetric systematic distortions of the distributions by experimental factors. This important question should be addressed by detailed simulations of the experiments, which the present author is not equipped to perform.

As it was observed in [6], when the event number is less than ~ 1000, the estimation becomes useless due to very large inaccuracies exceeding the above quoted unitarity bounds. The small branching ratio of the leptonic top decay renders dilepton events useless for our purposes. The importance of the event number to improve the statistics leads us to consider the abundant all-hadronic decays modes. We have shown that the price to pay is the loss of the measurement of c_{AV} and c_{AA}, but the increased event number can help to improve the accuracy of the rest of the parameters.

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A. Linear representation

In the linear representation there are no dimension 4 or 5 operators. In the following I provide a list of those operators with two quark fields, at dimension six, which in the unitary gauge generate vertices involving at least one top quark (terms proportional with light or b quark masses, and also all lepton fields, have been dropped):

$$\begin{aligned}
b_1^{\alpha\beta} &\times (\overline{q}_L^\alpha\, iD_\mu d_R^\beta)\, iD^\mu\Phi + h.c. \\
b_2^{\alpha\beta} &\times (\overline{q}_L^\alpha\, iD_\mu u_R^\beta)\, \epsilon\, \overline{iD^\mu\Phi} + h.c. \\
b_3^{\alpha\beta} &\times (\overline{q}_L^\alpha\, \gamma_\mu q_L^\beta)\, (\overline{\Phi}\, iD^\mu\Phi) + h.c. \\
b_4^{\alpha\beta} &\times (\overline{q}_L^\alpha\, \gamma_\mu \frac{\tau^a}{2} q_L^\beta)\, (\overline{\Phi}\, \frac{\tau^a}{2}\, iD^\mu\Phi) + h.c. \\
b_5^{\alpha\beta} &\times (\overline{u}_R^\alpha\, \gamma_\mu u_R^\beta)\, (\overline{\Phi}\, iD^\mu\Phi) + h.c. \\
b_6^{\alpha\beta} &\times (\overline{u}_R^\alpha\, \gamma_\mu d_R^\beta)\, (\Phi^T\epsilon\, iD^\mu\Phi) + h.c. \\
b_7^{\alpha\beta} &\times (\overline{q}_L^\alpha\, u_R^\beta\, \epsilon\overline{\Phi}^T)\, (\overline{\Phi}\Phi) + h.c. \\
g'\, b_8^{\alpha\beta} &\times \overline{q}_L^\alpha\, \sigma^{\mu\nu} B_{\mu\nu} u_R^\beta\; \epsilon\overline{\Phi}^T + h.c. \\
g\, b_9^{\alpha\beta} &\times \overline{q}_L^\alpha\, \sigma^{\mu\nu} u_R^\beta\; \frac{\tau^a}{2} V_{\mu\nu}^a\; \epsilon\overline{\Phi}^T + h.c. \\
g_s\, b_{10}^{\alpha\beta} &\times \overline{q}_L^\alpha\, \sigma^{\mu\nu}\; \frac{\lambda^a}{2} G_{\mu\nu}^a\; u_R^\beta\; \epsilon\overline{\Phi}^T + h.c. \\
g\, b_{11}^{\alpha\beta} &\times \overline{q}_L^\alpha\, \sigma^{\mu\nu} d_R^\beta\; \frac{\tau^a}{2} V_{\mu\nu}^a\; \Phi + h.c.
\end{aligned} \tag{A.1}$$

Here, α, β are flavor indices, q_L^α are the left handed quark doublets, u_R, d_R are the right-handed singlet quarks, $B_{\mu\nu}$ and $V_{\mu\nu}$ are the hypercharge and SU(2) field strengths, $G^a_{\mu\nu}$ is the gluon field strength, and ϵ is the antisymmetric 2×2 tensor in SU(2) space. The covariant derivative D_μ includes all electroweak and gluon fields. The coupling constants, b_i, are (not necessarily Hermitian) complex matrices. In the above expressions the equations of motion have been used. This is consistent if one keeps the 4-quark operators [9]. In the unitary gauge, replacing $\Phi \to (0, \frac{v+H}{\sqrt{2}})$ allows us to write down the new physics operators in a physical basis where Goldstone fields are absent.

There is a restriction on the size of b_7 from the fact that it gives mass to the quarks. The absence of fine tuning requires that this additional mass, $\frac{b_7^{\alpha\beta}}{2}\left(\frac{v}{\sqrt{2}}\right)^3$, does not significantly exceed the physical quark masses. This coupling, however, will not give a contribution to our process. The coupling b_4 contains a piece $-\frac{b_4^{\alpha\beta}}{2}\frac{g}{\sqrt{2}}\,\overline{u}_L^\alpha\gamma^\mu \times W_\mu d_L^\beta$ whose effect is to modify the corresponding CKM matrix element $V_{\alpha\beta}$. Again, the absence of fine tuning imposes a restriction on the size of b_4. There are obvious restrictions of the flavor changing operators in the list.

B. Nonlinear representation

In the nonlinear representation we have the following operators with two quark fields, again dropping those that contain no top quark:

- Dimension 4

$$-\frac{g}{2\cos\theta}\Big[\lambda^{\alpha\beta}_{ZLu} \times Z_\mu\,\overline{u}^\alpha_L\gamma^\mu u^\beta_L + \lambda^{\alpha\beta}_{ZLd} \times Z_\mu\,\overline{d}^\alpha_L\gamma^\mu d^\beta_L + \lambda^{\alpha\beta}_{ZRu} \times Z_\mu\,\overline{u}^\alpha_R\gamma^\mu u^\beta_R + \\ + \lambda^{\alpha\beta}_{ZRd} \times Z_\mu\,\overline{d}^\alpha_R\gamma^\mu d^\beta_R\Big] - \frac{g}{\sqrt{2}}\Big[\lambda^{\alpha\beta}_{WR} \times W_\mu\,\overline{u}^\alpha_R\gamma^\mu d^\beta_R + h.c.\Big] \quad \text{(B.1)}$$

  and $\lambda^{\alpha\beta}_{WL}$ is absorbed into the CKM matrix element $V_{\alpha\beta}$. Here, each λ_Z is Hermitian but λ_{WR} can be any complex matrix.

- Dimension 5
  The magnetic moments are

$$-\overline{u}^\alpha\sigma^{\mu\nu}\frac{\kappa^{\alpha\beta}_\gamma eF_{\mu\nu} + \kappa^{\alpha\beta}_g g_s\lambda^a/2G^a_{\mu\nu} + \kappa^{\alpha\beta}_Z g/(2\cos\theta)Z_{\mu\nu}}{4m_t}u^\beta - \\ - \left\{\overline{u}^\alpha\sigma^{\mu\nu}d^\beta\frac{\kappa^{\alpha\beta}_W g/\sqrt{2}W_{\mu\nu}}{4m_t} + h.c.\right\}, \quad \text{(B.2)}$$

  the electric moments are found from these with the replacement $\kappa \to \tilde{\kappa}, X_{\mu\nu} \to {}^*X_{\mu\nu}$.

The weak derivative couplings are

$$
\begin{aligned}
&-\frac{g}{2\cos\theta} Z^\mu \frac{\kappa_Z'^{\alpha\beta}}{2m_t} \overline{u}_L^\alpha \, i \overset{\leftrightarrow}{D}_\mu \, u_R^\beta - \\
&-\left\{ \frac{g}{\sqrt{2}} W^\mu \left(\frac{\kappa_{RW}'^{\alpha\beta}}{4m_t} \, \overline{u}_L^\alpha \, i \overset{\leftrightarrow}{D}_\mu \, d_R^\beta + \frac{\kappa_{LW}'^{\alpha\beta}}{4m_t} \, \overline{u}_R^\alpha \, i \overset{\leftrightarrow}{D}_\mu \, d_L^\beta \right) + h.c. \right\} .
\end{aligned}
\tag{B.3}
$$

Again, $\kappa_\gamma, \kappa_g, \kappa_Z, \tilde{\kappa}_\gamma, \tilde{\kappa}_g, \tilde{\kappa}_Z$ are Hermitian and $\kappa'_Z, \kappa'_{LW}, \kappa'_{RW}$ need not be. Because κ_W and $\tilde{\kappa}_W$ are not independent, $\tilde{\kappa}_W{}^* W_{\mu\nu} \overline{u}_R \sigma^{\mu\nu} d_L \equiv i\tilde{\kappa}_W W_{\mu\nu} \overline{u}_R \sigma^{\mu\nu} d_L$, we can require that κ_W and $\tilde{\kappa}_W$ be both Hermitian.

- dimension 6

  Dropping all operators that involve, in addition to two quark fields, at least two more electroweak bosons, we find for both $H = L, R$:

$$
\begin{aligned}
&\frac{g}{2\cos\theta} \frac{\rho_{HZ}^{\alpha\beta}}{m_t^2} \times \overline{u}_H^\alpha \gamma^\mu \, i \overset{\leftrightarrow}{D}{}^\nu \, u_H^\beta \, (\partial_\mu Z_\nu + \partial_\nu Z_\mu) \\
&\frac{g}{\sqrt{2}} \frac{\rho_{HW}^{\alpha\beta}}{m_t^2} \times \overline{u}_H^\alpha \gamma^\mu \, i \overset{\leftrightarrow}{D}{}^\nu \, d_H^\beta \, (\partial_\mu W_\nu + \partial_\nu W_\mu) \\
&\frac{g\, g_s}{2\cos\theta} \frac{\rho_{HZ}'^{\alpha\beta}}{m_t^2} \times \; Z_\mu \, \overline{u}_H^\alpha \, \frac{\lambda^a}{2} G_a^{\mu\nu} \, \gamma_\nu u_H^\beta \\
&\frac{g\, g_s}{\sqrt{2}} \frac{\rho_{HW}'^{\alpha\beta}}{m_t^2} \times \; W_\mu \, \overline{u}_H^\alpha \, \frac{\lambda^a}{2} G_a^{\mu\nu} \, \gamma_\nu d_H^\beta \; + h.c. \\
&\frac{g\, g_s}{2\cos\theta} \frac{\tilde{\rho}_{HZ}'^{\alpha\beta}}{m_t^2} \times \; Z_\mu \, \overline{u}_H^\alpha \, \frac{\lambda^a}{2} {}^* G_a^{\mu\nu} \, \gamma_\nu u_H^\beta \\
&\frac{g\, g_s}{\sqrt{2}} \frac{\tilde{\rho}_{HW}'^{\alpha\beta}}{m_t^2} \times \; W_\mu \, \overline{u}_H^\alpha \, \frac{\lambda^a}{2} {}^* G_a^{\mu\nu} \, \gamma_\nu d_H^\beta \; + h.c.
\end{aligned}
\tag{B.4}
$$

  Here, $\rho_{HZ}, \rho'_{HZ}, \tilde{\rho}'_{HZ}$ are Hermitian but $\rho_{HW}, \rho'_{HW}, \tilde{\rho}'_{HW}$ need not be.

Note that in the nonlinear representation the covariant derivative D_μ involves only the gluon and electromagnetic fields but not the W and Z. All combinations with these fields should be included as there is no restriction from EW gauge invariance.

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